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An open string landscape

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Abstract: The effect of fluxes on open string moduli is studied by analyzing the constraints imposed by supersymmetry on D-branes in type IIB flux backgrounds. We show that generically the conditions of supersymmetry cannot be maintained when moving along the geometrical moduli space of the brane, so that open string moduli are lifted. We argue that there is a disconnected and discrete set of supersymmetric solutions to the open string equations of motion, which extends the familiar closed string landscape to the open string sector.

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1. Introduction and conclusions

One of the lessons of the second superstring revolution is that there is a unique underlying theory, namely M-theory. Recent work on flux vacua [1 – 4], however, supports the view that the theory has a very large [5, 6] number of ground states,[1] generically dubbed as the landscape of string vacua [7]. Thus far, most of the analysis has focused on studying, for a given background geometry, the critical points of the potential as a function of the closed string moduli and flux quanta. It has been found that, generically, the complex structure closed string moduli are fixed by the flux-induced perturbative superpotential, while the Kähler moduli have flat directions which nevertheless may be lifted by non-perturbative effects.

In this paper we analyze the analogous problem for open string moduli in type IIB flux compactifications. We derive the conditions for D-branes to preserve supersymmetry, closely following [8]. Supersymmetry imposes constraints both on the geometrical properties of the cycle wrapped by the D-brane, and on the gauge invariant field strength $\mathcal{F}$ living on its worldvolume. In the absence of background fluxes, the open string moduli

[1] At least in the supergravity approximation in which they are found.

arise from the space of geometrical deformations that preserve the holomorphicity of the cycle, as well as from Wilson lines. In the presence of fluxes, we show that the geometrical moduli are generically lifted, while the Wilson lines remain flat directions. This parallels the closed string analysis, in which the perturbative superpotential generically fixes the complex structure moduli, but not the Kähler moduli.

From the viewpoint of the four-dimensional effective field theory description, the conditions imposed by supersymmetry on the open string moduli should be interpreted as arising from a flux-induced superpotential for the open string fields. We find that, generically, there is a discretum of supersymmetric open string vacua disconnected from each other, since D-branes can be placed in a supersymmetric way only at a discrete set of points. This increases the number of choices that must be made in order to specify a vacuum, but now the choices are in the open string sector. We have named this space of choices the open string landscape. It would be interesting to map out this sector in more detail and try to extract features that could be of phenomenological interest.

Our analysis may also shed light on the problem of closed string moduli stabilization. Supersymmetric D-brane instantons can generate a non-perturbative superpotential that stabilizes the volume modulus of the compactification manifold, which is left unfixed by the flux-induced perturbative part. Generation of a superpotential by one such instanton depends on the precise number of fermionic zero-modes on its worldvolume. Supersymmetry implies that a reduction in the number of bosonic zero-modes in the presence of fluxes must be accompanied by the corresponding reduction in the number of their fermionic counterparts. Therefore, an instanton that does not contribute in the absence of fluxes may now contribute to the superpotential.

The plan of the paper is as follows. In section 2 we employ the κ-symmetric formulation of D-branes to derive the conditions imposed on them by unbroken supersymmetry in the presence of fluxes. We express these conditions in terms of the geometry of the cycle wrapped by the brane and its worldvolume flux $\mathcal{F}$. In section 3 we show that, generically, the supersymmetry condition on $\mathcal{F}$ reduces the would-be moduli space associated to deformations of the cycle to a set of isolated points. We also propose an interpretation of the supersymmetry conditions in terms of a flux-generated superpotential for the open string moduli. In section 4 we illustrate the general analysis of section 3 with a simple model, in which the appearance of an open string landscape can be seen explicitly. We discuss some generic features of this new landscape of vacua. Section 5 contains some formal and phenomenological applications of our analysis. Some calculations are relegated to the appendix.

2. Supersymmetric D-branes in flux backgrounds

In this section, we determine the brane embeddings that preserve supersymmetry by doing a D-brane worldvolume analysis. In this approach, the supersymmetries preserved by a D-brane in a $\mathcal{N} \geq 1$ supergravity background are those generated by background Killing spinors η that satisfy the condition [9]

$$\Gamma_\kappa \eta = \eta \,, \tag{2.1}$$

where Γ_κ is the traceless, unit-square matrix that appears in the κ-symmetry transformation of the D-brane worldvolume fermions [10]. This approach has the benefit that it can be applied to any supergravity background, in particular to supersymmetric backgrounds with fluxes.

In this paper we will concentrate on type IIB Dp-branes, for which Γ_κ takes the form[2]

$$\Gamma_\kappa = \frac{\sqrt{|g|}}{\sqrt{|g+\mathcal{F}|}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{\mu_1\nu_1\ldots\mu_n\nu_n} \mathcal{F}_{\mu_1\nu_1}\ldots\mathcal{F}_{\mu_n\nu_n}(-1)^n(\sigma_3)^{n+\frac{p-3}{2}}\sigma_2 \otimes \Gamma_{(0)}\,, \tag{2.2}$$

where[3]

$$\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{g}}\,\epsilon^{\mu_1\cdots\mu_{(p+1)}}\gamma_{\mu_1\cdots\mu_{(p+1)}}\,. \tag{2.3}$$

In these formulas, $\mu, \nu = 0, 1, \ldots, p$ are worldvolume indices, g is the induced metric on the D-brane, and

$$\mathcal{F} = 2\pi\alpha' F + B \tag{2.4}$$

is the gauge-invariant field strength on the D-brane. Throughout this paper, a pull-back of spacetime forms onto the D-brane worldvolume is understood where necessary, such as that of the B-field in the equation above. The γ-matrices are induced from the spacetime Γ-matrices as

$$\gamma_\mu = \partial_\mu Z^M \Gamma_M\,, \tag{2.5}$$

where Z^M are local coordinates in spacetime.

Our present goal is to solve for the supersymmetric D-brane embeddings in warped type IIB flux backgrounds. It will be convenient to first present the analysis performed by Mariño, minasian, Moore, and Strominger (MMMS) in [8], where they analyze the embedding condition of D-branes in Calabi-Yau compactifications and in the presence of a closed B-field. As we will show, the conditions found in [8] can be easily generalized to the case of our interest after some simple modifications.

2.1 D-branes in a Calabi-Yau space with a closed B-field

In [8], a general analysis of the conditions implied by eq. (2.1) was performed for type II D-branes embedded in special holonomy manifolds and with a closed B-field. In particular, it was analyzed the case of $\mathcal{N} = 2$ type IIB compactifications on Calabi-Yau three-folds (CY_3), for which the spacetime metric takes the form

$$ds^2 = \delta_{ab}\,dX^a dX^b + 2G^{\mathrm{CY}}_{n\overline{m}}\,dZ^n dZ^{\overline{m}}\,, \tag{2.6}$$

where Z^m and $Z^{\overline{m}}$ are holomorphic and anti-holomorphic coordinates on CY_3. All the type IIB field strengths vanish in this background but the role of the closed B-field cannot be ignored since it plays an important role on D-branes, as can be seen from equation (2.2).

[2] In our conventions, the 32 supersymmetry generators of type IIB supergravity are grouped into a doublet of real Majorana-Weyl fermions. The Pauli matrices act on this doublet index.

[3] We work in Euclidean signature on the worldvolume and on target space. This allows us to study both instantonic and space-filling branes. For the case of lorentzian space-filling D-branes, one can trivially analytically continue back to lorentzian signature.

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The first step is to write down the background Killing spinor appearing in (2.1). In this background, the associated Killing spinors η are linear combinations of the two covariantly constant spinors of the CY_3, η_+ and η_-. These are complex conjugate to one another, are normalized such that $\eta_\pm^\dagger \eta_\pm = 1$, and satisfy the usual relation[4]

$$\Gamma_{\overline{m}}\, \eta_+ = 0\,, \qquad \Gamma_m\, \eta_- = 0\,. \tag{2.7}$$

The Kähler two-form and the holomorphic three-form of the CY_3 can be defined in terms of these spinors through the relations

$$\Gamma_{m\bar{n}}\, \eta_+ = i J_{m\bar{n}}\, \eta_+\,, \qquad \Gamma_{mnp}\, \eta_+ = \Omega_{mnp}\, \eta_-\,, \tag{2.8}$$

so that $dJ = d\Omega = 0$.

Using the equations above, it was shown in [8] that an Euclidean D$(2n-1)$-brane wrapped on a $2n$ cycle $\mathcal{S}_{2n}$ of CY_3 preserves supersymmetry if and only if certain equations in terms of Ω and J are satisfied. These equations can be written as

$$e^{iJ-\mathcal{F}}\Big|_{2n} = e^{i\theta}\, \frac{\sqrt{|g+\mathcal{F}|}}{\sqrt{|g|}}\, \mathrm{dvol}_{2n}\,, \tag{2.9}$$

$$\iota_m \Omega \wedge e^{iJ-\mathcal{F}}\Big|_{2n} = 0 \qquad m = 1,2,3\,, \tag{2.10}$$

where the subscript '$2n$' means that we keep only the form of degree $2n$ and ι_m denotes the interior contraction with $\partial/\partial Z^m$. Pull-backs of spacetime forms onto the worldvolume are again understood. Finally, $e^{i\theta}$ is a phase that parametrizes the embedding of a U(1) family of $\mathcal{N}=1$ algebras inside the bulk $\mathcal{N}=2$ superalgebra. The same result also applies to a D$(2n+3)$-brane that fills the four flat directions in (2.6) and wraps a $2n$ cycle in CY_3.

Equation (2.10) can be shown to be equivalent to the condition that [8]

$$\mathcal{S}_{2n} \text{ is holomorphic}\,, \qquad \mathcal{F}^{(2,0)} = 0\,. \tag{2.11}$$

2.2 D-branes in a warped, flux background

We would like now to consider D-branes in a class of supersymmetric type IIB backgrounds with the following spacetime metric

$$ds^2 = \Delta(Z)^{-1}\, \delta_{ab}\, dX^a dX^b + 2G_{n\overline{m}}\, dZ^n dZ^{\overline{m}}\,. \tag{2.12}$$

The background is now warped by $\Delta(Z)$, and the compactification manifold $\mathcal{M}_6$ does no longer posses SU(3) holonomy, but belongs to the more general class of SU(3)-structure manifolds (see, e.g., [11−13]). In addition, one may also consider background fluxes. As shown in [13], in any supersymmetric compactification on a SU(3)-structure manifold one can define an hermitean metric, a normalizable supersymmetry generator η_+, its complex conjugate η_-, and a couple of differential forms J and Ω such that the relations (2.8) still hold. One can then check that the computations carried out in [8] also apply to this case, and that the supersymmetry equations (2.9) and (2.10) still apply to this more general type IIB backgrounds with fluxes and/or torsion.[5]

[4] Unless stated explicitly, all Γ-matrices we write down are curved space ones.

[5] In $\mathcal{N}=1$ supersymmetric backgrounds, the phase θ in (2.9) is fixed by the Killing spinor of the background.

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The simplest example of these class of vacua is obtained by taking $\mathcal{M}_6$ to be a conformal Calabi-Yau, more precisely taking $G_{n\overline{m}} = \Delta(Z) G^{\mathrm{CY}}_{n\overline{m}}$. Supersymmetric solutions to this ansatz can be found when one allows a non-trivial self-dual five form flux F_5, as well as a $(2,1)$ and primitive imaginary-self-dual (ISD) three form flux G_3 on the internal space. In addition sources of negative tension must be included when $\mathcal{M}_6$ is compact. The presence of fluxes, both from the NS-NS and RR sector, can be taken into account in the computation of the supersymmetric D-brane embeddings (2.1). The RR fluxes do not appear directly in the form of the κ-symmetry projector (2.2). Nevertheless they enter in an important way in determing the Killing spinor of the corresponding background and thus affect the solution of (2.1). The NS-NS fluxes enter both indirectly in the form of the background Killing spinor and directly in the form of the projector (2.2).

The Killing spinor of the warped Calabi-Yau background with ISD fluxes has the same structure as the one considered in [8] modulo the extra constraint:[6]

$$\sigma_2 \otimes \Gamma_{0123} \eta = \eta \,, \tag{2.13}$$

which reflects the fact that we only have $\mathcal{N}=1$ supersymmetry. We note that the projection on the spinor is precisely the one produced by a D3-brane. Just like in the previous section, we can construct normalizable spinors $\eta_\pm$ on the internal manifold, as well as the Γ-matrices

$$\eta_\pm = \Delta^{-1/4}\, \eta^{\mathrm{CY}}_\pm \,, \qquad \Gamma_a = \Delta^{-1/2}\, \Gamma^{\mathrm{CY}}_a \,, \qquad \Gamma_m = \Delta^{1/2}\, \Gamma^{\mathrm{CY}}_m \,, \tag{2.14}$$

where the superscript 'CY' corresponds to the quantities in the underlying Calabi-Yau geometry. It is now straightforward to verify that the relations (2.7) and (2.8) still hold, with J and Ω related to the corresponding forms in the underlying CY geometry by

$$J = \Delta\, J^{\mathrm{CY}} \,, \qquad \Omega = \Delta^{3/2}\, \Omega^{\mathrm{CY}} \,. \tag{2.15}$$

Notice that J and Ω are no longer closed due to the warp factor dependence. It then follows that the conditions for an Euclidean D-brane to preserve supersymmetry in a conformal Calabi-Yau background take the form (2.9) and (2.10). The phase θ, however, is now fixed to $e^{i\theta} = -1$ by the form of the Killing spinor (2.13).

The same conditions hold for a D$(2n+3)$-brane wrapped on $\mathcal{S}_{2n}$. This is because for such a D-brane the κ-symmetry matrix factorizes as

$$\Gamma = \Gamma_{\mathcal{S}_{2n}}\, \Gamma_{0123} \otimes \sigma_2 \,, \tag{2.16}$$

where $\Gamma_{\mathcal{S}_{2n}}$ is the matrix associated to an Euclidean D-brane wrapped on $\mathcal{S}_{2n}$. Since the Killing spinor satisfies (2.13) the conditions for the space-filling branes (filling Minkowski space) take the same form as for the brane just wrapping $\mathcal{S}_{2n}$. In particular, we see that the supersymmetry conditions for a space-filling D7-brane and an Euclidean D3-brane wrapped on the same cycle $\mathcal{S}_4$ are identical.

To summarize, the conditions for a D-brane to preserve supersymmetry in a SU(3)-structure compactification, and in particular in a warped Calabi-Yau, take the same form as in the standard Calabi-Yau case. Note, however, that in general the forms J and Ω are no longer calibrating forms in $\mathcal{M}_6$, since they are not closed.

[6]Here Γ_{0123} refers to flat Γ-matrices. Recall that we are working in euclidean signature, thus $\Gamma_{0123}^2 = 1$.

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2.3 Wrapping D-branes

Let us now briefly discuss the various possibilities for type IIB D-branes wrapping $2n$-cycles of $\mathcal{M}_6$. The main differences with the cases without fluxes arise due to the fact that the background selects a special $\mathcal{N}=1$ supersymmetry generator and branes have to be compatible with it. In addition, we now have an H_3 flux, which can give rise to novel effects.

Two-cycles. This case applies to a D1 instanton or a space-filling D5-brane. The constraint (2.10) forces the two cycle $\mathcal{S}_2 \subset \mathcal{M}_6$ to be holomorphic. The condition (2.9) reads

$$J + i\mathcal{F} = i\,\frac{\sqrt{|g+\mathcal{F}|}}{\sqrt{|g|}}\,\mathrm{dvol}_{\mathcal{S}_2}\,. \tag{2.17}$$

The only possible solution of this equation is to consider a configuration where $\mathcal{F} \to \infty$ and $J|_{\mathcal{S}_2} \to 0$ simultaneously. Physically this corresponds to a situation where the size of the two-cycle is shrunk to zero size while the amount of flux through the vanishing cycle is scaled in such a way that the collapsed D5-brane has a non-vanishing tension, giving rise to a fractional D(-1)/D3-brane. This type of branes are ubiquitous near Calabi-Yau singularities such as orbifolds and conifolds, being consistent supersymmetric branes in the corresponding flux backgrounds.

As evidenced from this analysis only fractional branes are consistent with supersymmetry. This could have intuitively been expected by noting that the branes have to be compatible with the projection imposed on the Killing spinor of the background, which is that of a regular D3-brane and recalling that a finite size space-filling D5-brane is not compatible with that projection.

Four-cycles. Let us now turn to the case of a space-filling D7-brane or a D3-brane instanton wrapping a four-cycle on $\mathcal{M}_6$. We expect this case to be more involved and interesting than the previous one since, as has been shown in the literature, background fluxes can lift D7-brane geometrical moduli [14 − 17]. From the point of view of euclidean D3-branes, these lifted moduli translate into a four-dimensional instanton with less zero modes than its (unwarped) Calabi-Yau analogue. As a result, there may be new D3-brane instantons contributing to the four-dimensional superpotential. We would expect such effects to be visible from the supersymmetry equations.

The equations we derived demand that the four-cycle is a divisor $\mathcal{S}_4$ which is holomorphically embedded in $\mathcal{M}_6$. Equations (2.9) and (2.10) read in this case

$$\mathcal{F}^{(2,0)} = \mathcal{F}^{(0,2)} = 0\,, \tag{2.18}$$

$$J \wedge \mathcal{F} = 0\,, \tag{2.19}$$

so we recover that $\mathcal{F}$ must be a (real) primitive[7] (1,1)-form on $\mathcal{S}_4$.

This condition on $\mathcal{F}$ can like-wise be written as the anti-selfduality condition on the divisor $*_4\mathcal{F} = -\mathcal{F}$. The fact that supersymmetry requires a specific polarization of $\mathcal{F}$ can be physically motivated by the observation that an anti-selfdual field configuration on the

[7] By primitive we mean any p-form α such that $\alpha \wedge J = 0$.

worldvolume of a D7-brane induces D3-brane charge, which aligns with the supersymmetry broken by the background. Therefore, the supersymmetry conditions (2.18) and (2.19) mean nothing but that we need to consider D7-branes whose worldvolume bundles carry a D3-brane charge.

Wrapping the Calabi-Yau. The final case to consider is that of a space-filling D9-brane or a D5-brane wrapping the entire manifold $\mathcal{M}_6$. Equations (2.9) and (2.10) reduce in this case to

$$\frac{1}{3!}\mathcal{F}^3 - \frac{1}{2!}J^2 \wedge \mathcal{F} = \frac{\sqrt{|g+\mathcal{F}|}}{\sqrt{|g|}}\, \mathrm{dvol}_{\mathcal{M}_6}\,, \tag{2.20}$$

$$\frac{1}{3!}J^3 - \frac{1}{2!}J \wedge \mathcal{F}^2 = 0 \tag{2.21}$$

$$\mathcal{F}^{(2,0)} = 0\,. \tag{2.22}$$

These equations have solutions when $\mathcal{M}_6$ is non-compact or when $H_3 = 0$. If the model is compact, so that H_3 does not vanish in cohomology, $\mathcal{F}$ cannot be globally well defined because it must satisfy the Bianchi identity $d\mathcal{F} = H_3$. Thus, as is stands, the D-brane worldvolume theory is not consistent and suffers from a Freed-Witten anomaly [18, 19]. It would be interesting to find out under what conditions a globally defined $\mathcal{F}$ can be constructed, perhaps by adding extra sources of $\mathcal{F}$ along the proposal in [20].

3. The moduli space of D7-branes: local aspects

The goal of this section is to analyze the local structure of the moduli space of supersymmetric D7-branes in a flux compactification (2.12). Specifically, we will assume that a solution exists, and ask what the possible deformations of the divisor $\mathcal{S}_4 \subset \mathcal{M}_6$ wrapped by the D7-brane are. We will see that all geometric moduli are generically lifted with respect to the fluxless case.

The NS-NS and RR background fluxes H_3 and F_3 may be combined into the complexified three-form flux

$$G_3 = F_3 - \tau H_3\,, \tag{3.1}$$

where $\tau = C_0 + i/g_s$ is the type IIB axion-dilaton field, which we take to be constant for simplicity. These fluxes must satisfy the Bianchi identities $dH_3 = dF_3 = 0$, as well as the proper quantization conditions on three-cycles of $\mathcal{M}_6$. G_3 must be ISD, namely $*_6 G_3 = iG_3$, and hence harmonic. Supersymmetry further requires G_3 to be a primitive form (i.e., such that $G_3 \wedge J = 0$) of type (2,1) [21]. Since $H_3 = -\,\mathrm{Im}\, G_3/\,\mathrm{Im}\,\tau$, it follows that H_3 is a real three-form such that

$$H_3 \in H_p^{(2,1)} \oplus H_p^{(1,2)}\,, \tag{3.2}$$

where the subindex 'p' stands for primitive forms. A self-dual RR five-form flux is also present, but its precise form will not be needed here.

Consider now a supersymmetric D7-brane wrapping a four-cycle $\mathcal{S}_4$ of $\mathcal{M}_6$, with a worldvolume U(1) gauge field $\mathcal{F}$. By assumption, the four-cycle is holomorphic and $\mathcal{F}$ is anti-selfdual. We wish to determine the local moduli space for this D7-brane, namely the continuous deformations that respect these supersymmetry conditions. In the fluxless case, the moduli space would be parametrized by[8] [22]

$$\text{Geometric moduli}: \ \zeta^a \quad a = 1, \ldots, H^{(0,2)}(\mathcal{S}_4)\,, \tag{3.3}$$

$$\text{Wilson line moduli}: \ \xi^b \quad b = 1, \ldots, H^{(0,1)}(\mathcal{S}_4)\,. \tag{3.4}$$

The geometric moduli parametrize deformations that preserve the holomorphicity of the cycle. We will now examine under what circumstances such deformations can respect the anti-selfduality condition on $\mathcal{F}$.

We begin by recalling that in order for the gauge theory on the D7-brane to be anomaly-free, the cohomology class in $H^3(\mathcal{S}_4)$ defined by the pull-back of H_3 must vanish [18]; note that this condition is invariant under continuous deformations of the cycle. Under these circumstances, the full content of the Bianchi identity

$$d\mathcal{F} = H_3 \tag{3.5}$$

is to imply that $\mathcal{F}$ is a globally well defined form on $\mathcal{S}_4$. By the Hodge decomposition theorem, we may uniquely decompose it as

$$\mathcal{F} = h + d\xi + d^\dagger \eta\,, \tag{3.6}$$

where h is harmonic [23]. The anti-selfduality condition then translates into $d\xi = - *_4 d^\dagger \eta$ and $h^{\rm SD} = 0$, where $h^{\rm SD}$ is the self-dual component of h. One solution of the first equation is $\xi = *_4 \eta$, which may always be achieved by a convenient choice of the worldvolume gauge potential A in (2.4).

We are thus left with a non-trivial constraint only on the variation under a holomorphic deformation of the harmonic part of $\mathcal{F}$. To examine this, let us expand it as

$$\mathcal{F}^{\rm har} = \sum_{i=1}^{h^{0,2}} a_i(\zeta)\, \alpha^i + \text{c.c.} + \sum_{j=1}^{h_p^{1,1}} b_j(\zeta)\, \beta^j + c(\zeta)\, \gamma\,, \tag{3.7}$$

where α^i, β^j and γ are a basis of harmonic two-forms belonging to the middle cohomology of $\mathcal{S}_4$ as shown in table 1. The fact that $\mathcal{S}_4$ is a four-dimensional conformally Kähler submanifold implies that there is a unique cohomology class of harmonic (1,1) non-primitive forms, whose harmonic representative $\tilde{\gamma}$ is proportional to $J^{\rm CY}$. It also implies that $\alpha^i, \bar{\alpha}^i$ and γ are self-dual, whereas β^j are anti-selfdual. The coefficients a^i, b^j, and c^j may depend on the positions ζ^a of the four-cycle, but not on the choice of Wilson lines ξ^b, since $\mathcal{F}$ is independent of these. Without loss of generality, we take the initial $\mathcal{S}_4$ to correspond to $\zeta = 0$. Since this is supersymmetric by assumption, we have $a^i(0) = c(0) = 0$.

[8]More precisely, ζ^a and ξ^b should be thought of as massless chiral multiplets transforming in the adjoint representation of the gauge group of the D7-brane. They may or may not be true moduli, as there may exist a non-trivial superpotential W for these fields with no quadratic terms.

$$[\bar{\alpha}^i] \in H^{2,0} \quad [\beta^j] \in H^{1,1}_p \quad [\alpha^i] \in H^{0,2}$$
$$[\gamma] \in H^{1,1}_{np}$$

Table 1: Middle cohomology of $\mathcal{S}_4$ and harmonic representatives.

Let $\tilde{\alpha}_i$, $\tilde{\beta}_j$ and $\tilde{\gamma}$ be sets of harmonic dual forms, such that

$$\int_{\mathcal{S}_4} \tilde{\alpha}_i \wedge \alpha^j = \delta_i^j\,, \qquad \int_{\mathcal{S}_4} \tilde{\beta}_i \wedge \beta^j = \delta_i^j\,, \qquad \int_{\mathcal{S}_4} \tilde{\gamma} \wedge \gamma = 1\,. \tag{3.8}$$

Then

$$a_i = \int_{\mathcal{S}_4} \tilde{\alpha}_i \wedge \mathcal{F}^{\rm har} = \int_{\mathcal{S}_4} \tilde{\alpha}_i \wedge \mathcal{F}\,, \tag{3.9}$$

where in the last equation we used the fact that $\tilde{\alpha}_i$ is harmonic. In fact, the $\tilde{\alpha}_i$ may be constructed as follows. An infinitessimal deformation of $\mathcal{S}_4$ that preserves its holomorphicity is specified by a holomorphic section X of the normal bundle of $\mathcal{S}_4$. Let X_i be a basis of such sections. Then the pull-back of $\iota_{X_i}\Omega^{\rm CY}$ is a basis of harmonic (2,0) forms on $\mathcal{S}_4$, and this map provides an isomorphism $\tilde{\alpha}_i = \iota_{X_i}\Omega^{\rm CY}$ between holomorphic deformations of $\mathcal{S}_4$ and $H^{2,0}(\mathcal{S}_4)$. In this basis, equation (3.9) becomes

$$a_i(\mathcal{S}_4) = \int_{\mathcal{S}_4} \iota_{X_i}\Omega^{\rm CY} \wedge \mathcal{F}\,, \tag{3.10}$$

which vanishes identically if the cycle is holomorphic and the supersymmetry condition (2.10) is satisfied, as expected. Analogously, the condition (2.19) directly implies that the coefficient $c(\mathcal{S}_4)$ vanishes.

Let us now determine how these coefficients vary as we deform the four-cycle. For any two four-cycles $\mathcal{S}_4$ and $\mathcal{S}_4'$ related by a continuous deformation there exists a five-chain Σ_5 such that $\partial\Sigma_5 = \mathcal{S}_4' - \mathcal{S}_4$. Stokes' theorem and the Bianchi identity (3.5) imply that

$$a_i(\mathcal{S}_4') - a_i(\mathcal{S}_4) = \int_{\Sigma_5} \iota_{X_i}\Omega^{\rm CY} \wedge H_3 + d(\iota_{X_i}\Omega^{\rm CY}) \wedge \mathcal{F}\,, \tag{3.11}$$

where $\mathcal{F}$ is now the natural extension of the worldvolume flux to the five chain Σ_5. Notice that (3.11) identically vanishes whenever $\mathcal{F}$ comes from the pull-back of a closed (1,1) B-field.

The coefficients b_j and c are given by formulas analogous to (3.9) with $\tilde{\alpha}_i$ replaced by $\tilde{\beta}_j$ and $\tilde{\gamma}$, respectively. Notice that Stokes' theorem now implies that c is constant under holomorphic deformations, since

$$c(\mathcal{S}_4') - c(\mathcal{S}_4) = \int_{\Sigma_5} J^{\rm CY} \wedge H_3 = 0 \tag{3.12}$$

by virtue of the facts that $\tilde{\gamma}$ can be taken to be the pull-back of $J^{\rm CY}$ and H_3 is primitive.

The coefficients b_j will generically change under the deformation $\mathcal{S}_4 \to \mathcal{S}_4'$, but recall that the forms β^j are anti-selfdual and hence compatible with supersymmetry. We thus conclude that the conditions for a holomorphic deformation of $\mathcal{S}_4$ to preserve supersymmetry are

$$a_1(\zeta^1, \ldots, \zeta^{h^{0,2}}) = 0\,,$$
$$\vdots \tag{3.13}$$
$$a_{h^{0,2}}(\zeta^1, \ldots, \zeta^{h^{0,2}}) = 0\,.$$

We will see below that the a_i are holomorphic functions of the deformations $\{\zeta^i\}$. Hence, (3.13) defines (locally) a complex submanifold within the moduli space of $\mathcal{S}_4$ that will be denoted as $\mathcal{M}^F(\mathcal{S}_4) \subset \mathcal{M}(\mathcal{S}_4)$. Intuitively, the codimension of $\mathcal{M}^F(\mathcal{S}_4)$ inside $\mathcal{M}(\mathcal{S}_4)$ equals the number of geometrical moduli lifted by the presence of the flux H_3. Notice that (3.13) is a system of $h^{0,2}(\mathcal{S}_4)$ equations for $h^{0,2}(\mathcal{S}_4)$ unknowns. Therefore, generically the solution will consist of a set of isolated points, and so all the geometrical moduli of the D7-brane will be lifted.

The number of infinitessimal deformations around a given point is $h^{0,2} - r$, where r is the rank of the matrix

$$\frac{\partial a_i}{\partial \zeta^j} = \int_{\mathcal{S}_4} L_{X_j}\left(\iota_{X_i}\Omega^{\text{CY}} \wedge \mathcal{F}\right) = \int_{\mathcal{S}_4} L_{X_j}\left(\iota_{X_i}\Omega^{\text{CY}}\right) \wedge \mathcal{F} + \iota_{X_i}\Omega^{\text{CY}} \wedge \iota_{X_j} H_3\,. \tag{3.14}$$

Here we have used the fact that ζ^i may be though as the components of a holomorphic section X in a given basis $\{X_i\}$, namely $X = \zeta^i X_i$. Notice that $L_{X_j}\iota_{X_i}\Omega^{\text{CY}} = \iota_{[X_j,X_i]}\Omega + \iota_{X_i} d\iota_{X_j}\Omega$ is again a (2,0)-form, where L_{X_j} stands for the Lie derivative with respect to the holomorphic deformation represented by X_j. Hence the r.h.s. of (3.14) simplifies once we impose the supersymmetry condition (2.18), and we end up with

$$\frac{\partial a_i}{\partial \zeta^j} = \int_{\mathcal{S}_4} \iota_{X_i}\Omega^{\text{CY}} \wedge \iota_{X_j} H_3 = \frac{i g_s}{2} \int_{\mathcal{S}_4} \iota_{X_i}\Omega^{\text{CY}} \wedge \iota_{X_j} \bar{G}_3\,, \tag{3.15}$$

where the last equality follows from $H_3 = -\operatorname{Im} G_3 / \operatorname{Im}\tau$ and the fact that G_3 is a (2,1)-form. In particular, this implies that $(\partial a_i / \partial \bar{\zeta}^{\bar{j}}) = 0$, so the coefficients a_i are indeed holomorphic functions of $\{\zeta^i\}$.[9]

From the viewpoint of the four-dimensional effective field theory, the conditions for unbroken supersymmetry should arise from an effective superpotential $\mathcal{W}$ for the open string moduli. A natural interpretation is to identify the equations $a_i(\xi) = 0$ with $\partial_i \mathcal{W} = 0$, since both are holomorphic functions of the moduli. Note that these are indeed the equations for unbroken supersymmetry if the potential is of no-scale type. It would be interesting to verify this proposal in explicit examples.

4. The moduli space of D7-branes: global aspects

As we have shown, κ-symmetry provides us with a powerful tool to analyze the moduli space of a D7-brane in a flux background. Our analysis so far, however, has only provided us with a local description of the moduli space. Indeed, we have seen that for a D7-brane

[9] Poincaré invariance of the ten-dimensional background along the four non-compact directions also allows a (0,3) component of G_3, which breaks supersymmetry. In this case the coefficients a_i need no longer be holomorphic functions.

wrapping a supersymmetric four-cycle $\mathcal{S}_4$, a background flux G_3 reduces its geometrical moduli space to a subvariety $\mathcal{M}^F(\mathcal{S}_4) \subset \mathcal{M}(\mathcal{S}_4)$. Generically all the geometrical moduli are lifted, so $\mathcal{M}^F(\mathcal{S}_4)$ will locally be an isolated point of $\mathcal{M}(\mathcal{S}_4)$. However, the global structure of $\mathcal{M}^F(\mathcal{S}_4)$ may be more involved, and it may consist of a discrete set of points inside $\mathcal{M}(\mathcal{S}_4)$. One of the main purposes of the present section is to show that this will be usually the case.

It is clear that these global issues are of central interest for the statistical analysis of flux vacua in string theory [6], once that open strings are introduced in such analysis. Indeed, we will argue that including D7-branes in flux compactifications opens up a new landscape of possibilities for the type IIB discretum. Moreover, this open string landscape is directly connected to the physics of $D = 4$ non-abelian gauge theories and chiral matter, so any statistical prediction derived from it should have a clear interpretation in terms of low energy particle physics.

4.1 A toroidal example

Before performing a general analysis, let us illustrate both local and global features of the moduli space of D7-branes by considering an explicit flux compactification. For simplicity, we will consider a background such that constant fluxes and flat space are solution of the supergravity equations of motion. The simplest compact spaces of this kind are given by $\mathbf{T}^6$ or toroidal orbifolds such as $\mathbf{T}^4/\mathbb{Z}_2 \times \mathbf{T}^2$, $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [4, 24−26, 20]. Since these backgrounds are compact, the ansatz (2.12) requires sources of negative tension in the construction, such as O3$^-$-planes. Hence, we need to further mod out these toroidal backgrounds by $\Omega^{ws}\mathcal{R}(-1)^{F_L}$, where Ω^{ws} is the worldsheet parity operator and $\mathcal{R}$ acts by $(z^1, z^2, z^3) \mapsto (-z^1, -z^2, -z^3)$. For the sake of definiteness, let us choose the metric on $\mathbf{T}^6$ to be of the form $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$, and with background fluxes

$$F_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dy^3 + dy^1 \wedge dy^2 \wedge dy^3\right) \tag{4.1}$$

$$H_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3\right), \tag{4.2}$$

where $0 \leq x^i, y^i \leq 1$ and $N \in \mathbb{Z}$.[10]

The fluxes (4.1), (4.2) generate a superpotential

$$W = \int \Omega \wedge G_3 \propto (1 + \tau_1\tau_2) \cdot (1 + \tau_3\tau), \tag{4.3}$$

τ_i being the complex structure of the i^{th} $\mathbf{T}^2$ factor, and τ the complex dilaton-axion field. The supersymmetric minima of W are given by

$$\tau_1\tau_2 = -1, \qquad \tau_3\tau = -1 \tag{4.4}$$

which means that for this choice of fluxes only two combinations of complex structure moduli/dilaton are stabilized.

[10] In general, the integer N cannot take an arbitrary value, but needs to be of the form $N = nN_{\min}$. Here $n \in \mathbb{Z}$ and $N_{\min} \in \mathbb{N}$ depends on the homology of three-cycles in $\mathbf{T}^6/\Gamma$, as well as on the O3-plane content on the quotient space. For compactifications where only the usual O3$^-$-planes appear, we have that $N_{\min} = 2$ for $\Gamma = Id.$ [24] and that $N_{\min} = 4, 8$ for $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$, depending on the choice of discrete torsion [26, 20]. Roughly speaking, $N_{\min}$ indicates the minimum amount of flux that we can turn on.

We now want to introduce D7-branes in our compactification. As we know, these D7-branes must wrap holomorphic four-cycles $\mathcal{S}_4$ and the pull-back $H_3|_{\mathcal{S}_4}$ must be trivial in cohomology. The simplest possibility is to consider a D7-brane wrapping two complex dimensions $(\mathbf{T}^2)_j \times (\mathbf{T}^2)_k \subset (\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$ and being pointlike in $(\mathbf{T}^2)_i$, $j \neq i \neq k$. Following the type IIB model building literature, we will name such D7-brane as D7_i (see figure 1 below for an illustration of a D7_1-brane). Each of these D7-branes has the topology of $\mathbf{T}^4$, and hence its middle cohomology is given by $h^{2,0} = h^{0,2} = 1$, $h^{1,1}_p = 3$ and $h^{1,1}_{np} = 1$. As a consequence, each will have one geometrical modulus, which we will denote by ζ^i, which parametrizes the motion along $(\mathbf{T}^2)_i$.

The presence of a non-trivial H_3 on $\mathbf{T}^6$ gives rise to a non-closed B-field, which eventually will contribute to the gauge invariant two-form $\mathcal{F}$ on the D7-branes. Let us see how this happens for the D7_i-branes in our example. We will first consider a D7_1-brane. A suitable choice of gauge for the B-field in this case is given by

$$B = 4\pi^2\alpha' N \left(x^1 dx^2 \wedge dx^3 + y^1 dy^2 \wedge dx^3\right) \tag{4.5}$$

which is well-defined on the patch $\left(\{0 \leq x^1 < 1\} \times \{0 \leq y^1 < 1\}\right) \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$ inside $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$. Notice that this choice satisfies $B_{\mu\nu}(-x^i, -y^i) = -B_{\mu\nu}(x^i, y^i)$, compatible with the action $\mathcal{R}B = -B$ of the orientifold group on the B-field. In addition, (4.5) is globally well-defined over the four-cycle wrapped by the D7_1-brane, i.e., $\{\zeta^1\} \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$. Hence, in order to compute its pull-back we only need to consider this single patch, and we obtain

$$\begin{aligned} B|_{D7_1} &= 4\pi^2\alpha' N \left(x^1 dx^2 \wedge dx^3 + y^1 dy^2 \wedge dx^3\right) \\ &= 4\pi^2\alpha' N \,\frac{i}{2} \left(\frac{\tau_2}{\operatorname{Im}\tau_2} z^1 d\bar{z}^{\bar{2}} + \frac{\tau_1}{\operatorname{Im}\tau_1} \bar{z}^{\bar{1}} dz^2 \right) \wedge \frac{\operatorname{Im}(\bar{\tau}_3 dz^3)}{\operatorname{Im}\tau^3}\,, \end{aligned} \tag{4.6}$$

where we have made use of the relations (4.4). Notice that $B|_{D7_1}$ is a harmonic form, which is consistent with the fact that $H_3|_{D7_1}$ identically vanishes.

It is it easy to see that the $(0,2)$ component of $B|_{D7_1}$ depends holomorphically on z^1, which is nothing but the D7_1-modulus ζ^1. Indeed, in terms of the general expression (3.7), (4.6) translates into

$$a_1 = -(4\pi^2\alpha')\, 4N\, \zeta^1\, \tau_2\tau_3 \tag{4.7}$$

$$b_1 = (4\pi^2\alpha')\, 4N\, \operatorname{Re}\left(\zeta^1\, \tau_2\bar{\tau}_3\right) \tag{4.8}$$

$$b_2 = -(4\pi^2\alpha')\, 4N\, \operatorname{Im}\left(\zeta^1\, \tau_2\bar{\tau}_3\right) \tag{4.9}$$

$$b_3 = 0 \tag{4.10}$$

$$c = 0\,, \tag{4.11}$$

where we have identified ζ^1 with z^1. Here b_1 and b_2 correspond to the coefficients of the real forms $\operatorname{Re}(dz^2 \wedge d\bar{z}^{\bar{3}})/\operatorname{Im}\tau_2 \operatorname{Im}\tau_3$ and $\operatorname{Im}(dz^2 \wedge d\bar{z}^{\bar{3}})/\operatorname{Im}\tau_2 \operatorname{Im}\tau_3$, respectively. Notice that the fact that $c = 0$ was expected from the general results of section 3.

It is now clear that B will induce a non-vanishing $(2,0) + (0,2)$ component on the D7_1-brane, unless we choose the specific location $\zeta^1 = 0$ for it. We then conclude that the geometrical modulus ζ^1 of D7_1-branes is lifted after we introduce the fluxes (4.1) and (4.2).

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A similar result is obtained for D7_2-branes in the same background. The case of D7_3-branes, however, is different. Indeed, the pull-back of the B-field on $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times \{\zeta^3\}$ is given by

$$\begin{aligned} B|_{D7_3} &= 4\pi^2\alpha' N \frac{\mathrm{Im}\,\bar{\tau}_3\zeta^3}{\mathrm{Im}\,\bar{\tau}_3} \left(dx^1 \wedge dx^2 + dy^1 \wedge dy^2\right) \\ &= 4\pi^2\alpha' N \frac{\mathrm{Im}\,\bar{\tau}_3\zeta^3}{2i\,\mathrm{Im}\,\bar{\tau}_3} \left(\frac{\tau_2}{\mathrm{Im}\,\tau_2} dz^1 \wedge d\bar{z}^{\bar{2}} + \frac{\tau_1}{\mathrm{Im}\,\tau_1} d\bar{z}^{\bar{1}} \wedge dz^2\right), \end{aligned} \tag{4.12}$$

so that we find that $B|_{D7_3}$ defines a primitive (1,1)-form regardless of the value of ζ^3. That is, H_3 induces a supersymmetric B-field on $D7_3$, independently of its position and, as a result, no geometrical moduli are stabilized for D7_3-branes.

Notice that these results match exactly with those obtained in [16], where the effect of G_3 flux was computed on the effective theory of a D7-brane wrapping a $\mathbf{T}^4$. Again using (4.4), we can write the type IIB flux $G_3 = F_3 - \tau H_3$ obtained from (4.1) and (4.2) as

$$G_3 = 4\pi^2\alpha' N \frac{1}{2i} \left(\frac{\tau_2}{\mathrm{Im}\,\tau_2} dz^1 \wedge d\bar{z}^{\bar{2}} + \frac{\tau_1}{\mathrm{Im}\,\tau_1} d\bar{z}^{\bar{1}} \wedge dz^2\right) \wedge \frac{dz^3}{\tau_3} \tag{4.13}$$

and it is easy to see that the nonvanishing components of G_3 in terms of SU(3) irreducible representations are given by $S_{\bar{1}\bar{1}}$ and $S_{\bar{2}\bar{2}}$, as defined in [21]. By the results of [16], these are the flux components which give mass to the geometrical moduli of D7_1 and D7_2-branes, respectively, whereas they do not affect the D7_3-brane modulus.

In fact, one can be more precise and compute the supersymmetric masses of the D7-brane moduli ζ^1 and ζ^2, again using the techniques of section 3. In order not to digress from the present discussion, we will postpone such analysis to section 5.

A discrete set of $\mathcal{N} = 1$ D7-branes. So far we have used a toroidal example to illustrate the general discussion of section 3. However, being a compact model it turns out to be also useful to describe global aspects of the D7-brane moduli space. For instance, since we found that D7_1-brane geometrical moduli are lifted, one may wonder if there is only one or several values of ζ^1 allowed by supersymmetry. We know that there is at least one, given by $\zeta^1 = 0$, but in general there may be a discrete set of solutions.

Let us illustrate this fact with the choice of fluxes above. Notice that in the discussion above we have implicitly assumed that $\mathcal{F}_{D7} = B|_{D7}$. That is, we have set $F = dA = 0$ on (2.4). In general, we must consider the gauge invariant quantity $\mathcal{F}_{D7_1} = 2\pi\alpha' F_{D7_1} + B|_{D7_1}$, whose components in this case are given by

$$B|_{D7_1} = 4\pi^2\alpha' N \left(\zeta^1_x\, dx^2 \wedge dx^3 + \zeta^1_y\, dy^2 \wedge dx^3\right) \tag{4.14}$$

$$F_{D7_1} = 2\pi \left(n_1\, dx^2 \wedge dx^3 + n_2\, dy^2 \wedge dx^3 + \cdots\right), \tag{4.15}$$

where have parametrized the moduli space of the D7_1-brane as $\zeta^1 = \zeta^1_x + \tau_1\zeta^1_y$ for convenience. Here n_1, n_2 are integer numbers and '. . . ' stands for extra components of F which will not be directly relevant in the following, and we will assume them to vanish. Notice that both F_{D7_1} and $B|_{D7_1}$ are harmonic forms on $\{\zeta^1\} \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3$, but that only F_{D7_1} needs to satisfy the Dirac quantization condition.

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Now, in order to look for supersymmetric solutions for a $D7_1$-brane, we can search for choices of $B|_{D7_1}$ and F_{D7_1} which give us a vanishing $\mathcal{F}_{D7_1}$.[11] Since $\mathcal{F}_{D7_1}$ is given by the combination (2.4), it is clear that such cancellation will not be possible for arbitrary values of (ζ_x^1, ζ_y^1). The only possibilities are given by

$$\begin{aligned} \zeta_x^1 N &= -n_1 \in \mathbb{Z} \\ \zeta_y^1 N &= -n_2 \in \mathbb{Z} \end{aligned} \tag{4.16}$$

which indeed give us a discretum of possibilities. Each of these represents a $D7_1$-brane such that $\mathcal{F}_{D7_1}$ vanishes, and is hence trivially supersymmetric.

As an example, let us consider the background $\mathbf{T}^6/\Omega^{ws}\mathcal{R}(-1)^{F_L}$, such that there are 64 $O3^-$-planes in our compactification. The positions of such O3-planes, or more precisely their projection onto $(\mathbf{T}^2)_1$, are given by $(\zeta_x^1, \zeta_y^1) = \{(0,0), (0, \frac{1}{2}), (\frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})\}$. Each of these points is actually equivalent from the point of view of the compactification. In particular, since we have obtained that the origin $(\zeta_x^1, \zeta_y^1) = (0,0)$ is a supersymmetric location for a $D7_1$-brane, so must be the equivalent points $(0, \frac{1}{2}), (\frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})$. Now, by looking at the general solution (4.16) we see that these four points will be supersymmetric locations if and only if $N \in 2\mathbb{Z}$. This may seem quite a restrictive set of fluxes at first sight but, as pointed out in [24], $N \in 2\mathbb{Z}$ are indeed the allowed choices of flux for this orientifold background. It is amusing that we have rephrased the flux quantization conditions of [24] in terms of translational symmetries of the background $\mathbf{T}^6/\Omega^{ws}\mathcal{R}(-1)^{F_L}$.

4.2 The open string landscape

The toroidal example above teaches us an interesting lesson. As we have seen in section 3, the geometrical moduli of a D7-brane are lifted by the presence of the background flux. This reduces its geometrical moduli space to a subvariety $\mathcal{M}^F(D7) \subset \mathcal{M}(D7)$ which, generically, is zero-dimensional.[12] Locally this zero dimensional space looks like a point, but globally this need not be the case. Indeed, above we have shown that the former moduli space of a $D7_1$-brane, which is nothing but $\mathcal{M}(D7_1) = (\mathbf{T}^2)_1$, is now replaced by a grid of points or two-dimensional lattice $\mathcal{M}^F(D7_1) = \Lambda_2$, as illustrated in figure 1.

This lattice of supersymmetric $D7_1$-branes is given by (4.16), which can be expressed as

$$N\zeta^1 = m_x + \tau_1 m_y, \quad m_x, m_y \in \mathbb{Z} \tag{4.17}$$

and which generalizes the previous solution $\zeta^1 = 0$, obtained from a local analysis. Since ζ^1 parametrizes the moduli space of $D7_1$-branes, the lattice Λ_2 can be expressed as

$$\Lambda_2 = \left\{ \frac{m_x + \tau_1 m_y}{N} \middle| m_x, m_y \in \mathbb{Z} \right\} / \Lambda \quad \subset \quad (\mathbf{T}^2)_1 , \tag{4.18}$$

where $\Lambda = \{n_x + \tau_1 n_y | n_x, n_y \in \mathbb{Z}\}$ is the usual lattice of identifications that compactifies $\mathbb{C}$ to $(\mathbf{T}^2)_1$.

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[11] Actually, we should be looking for combinations such that $\mathcal{F}_{D7_1}^{(2,0)}$ vanishes, which is a milder condition. However, it does not give new solutions in the case at hand.

[12] Notice that in all this discussion we are neglecting the moduli space of Wilson lines on each D7-brane. Since in principle they are not affected by the background flux G_3, their contribution to the moduli space of the D7-brane would remain untouched.

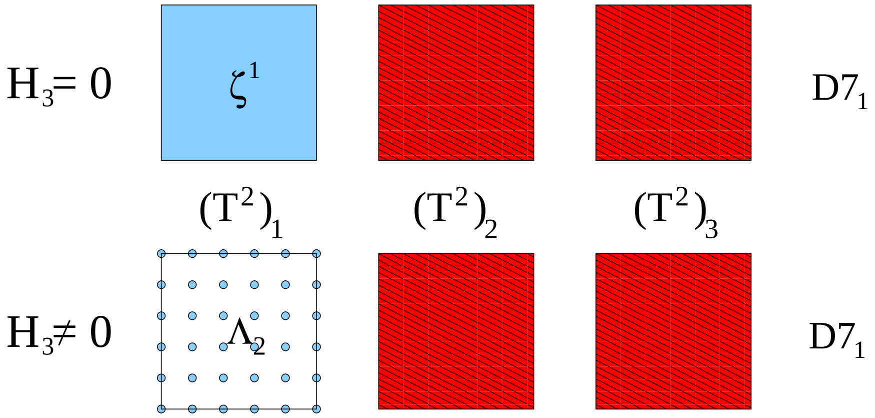

Figure 1: Open string landscape of a D7_1-brane. Before the flux H_3 is introduced, the geometrical moduli space of a D7_1-brane is the $\mathbf{T}^2$ where it is point-like, i.e., $(\mathbf{T}^2)_1$. After the flux H_3 is turned on, this geometrical moduli space is restricted to a lattice of points $\Lambda_2 \subset (\mathbf{T}^2)_1$.

We can now attempt to extend these results to more general flux compactifications. Let us consider type IIB compactified on $\mathcal{M}_6$, and containing a supersymmetric D7-brane wrapped on $\mathcal{S}_4$. In absence of fluxes or torsion, the geometrical moduli space of such D7-brane would be given by $\mathcal{M}(\mathcal{S}_4)$, parametrized by $\{\zeta^1, \ldots, \zeta^{h^{0,2}}\}$.[13] When we introduce the background flux G_3, the geometrical moduli space of the D7-brane should be reduced to $\mathcal{M}^F(\mathcal{S}_4)$. Locally, this reduced moduli space would look like a submanifold $\mathcal{M}^f(\mathcal{S}_4)$ defined by imposing the equations (3.13) on the former moduli space. Globally, however, we would expect a lattice of such $\mathcal{M}^f(\mathcal{S}_4)$'s. Schematically, if the fluxes stabilize q complex geometrical moduli of the D7-brane we would expect something of the form

$$\mathcal{M}^F(\mathcal{S}_4) \;=\; \Lambda_{2q} \times \mathcal{M}^f(\mathcal{S}_4)\,, \tag{4.19}$$

where $\mathcal{M}^f(\mathcal{S}_4)$ has complex dimension $h^{0,2} - q$, and is transverse to the $2q$-dimensional lattice $\Lambda_{2q} \subset \mathcal{M}(\mathcal{S}_4)$.[14] For a generic choices of fluxes, we would expect all the geometrical moduli of the D7-brane to be lifted, so that $q = h^{0,2}$ and $\mathcal{M}^F(\mathcal{S}_4) = \Lambda_{2h^{0,2}}$ is a lattice of points inside $\mathcal{M}(\mathcal{S}_4)$.

Whereas the above picture works at the intuitively level, it is important to realize that the D-brane supersymmetry conditions provide us with a precise definition of the new moduli space $\mathcal{M}^F(\mathcal{S}_4)$. In the present context this definition would read

$$\mathcal{M}^F(\mathcal{S}_4) \;=\; \left\{\mathcal{S}_4 \in [\mathcal{S}_4] \,\middle|\, B|_{\mathcal{S}_4} \in A_p^{(1,1)}(\mathcal{S}_4, \mathbb{R}) \oplus H^2_p(\mathcal{S}_4, \mathbb{Z})\right\}, \tag{4.20}$$

which means that $B|_{\mathcal{S}_4}$ can either be a primitive real (1,1)-form of $\mathcal{S}_4$ ($A_p^{(1,1)}(\mathcal{S}_4, \mathbb{R})$), a integral primitive two-form ($H^2_p(\mathcal{S}_4, \mathbb{Z})$), or a combination of both. Intuitively, if $B|_{\mathcal{S}_4}$ is

[13] In the following, we are implicitly assuming that $\mathcal{M}(\mathcal{S}_4)$ is a smooth manifold of definite dimension in order to simplify our discussion. Our statements should be easy to generalize to more involved cases.

[14] More likely, (4.19) would not be a direct product, but the geometry and topology of $\mathcal{M}^f(\mathcal{S}_4)$ would vary for each lattice point of Λ_{2q}.

(1,1) and primitive it does not break supersymmetry, and if it has a component on $H^2_p(\mathcal{S}_4,\mathbb{Z})$ it can be cancelled with the appropriate gauge bundle $F = dA$ such that $\mathcal{F} = 2\pi\alpha' F + B$ is again a real primitive (1,1)-form.

This definition turns out to be quite useful in the generic case where $\mathcal{M}^F(\mathcal{S}_4)$ is the $2h^{(0,2)}$-dimensional lattice $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$. Indeed, to each point $l \in \Lambda_{2h^{0,2}(\mathcal{S}_4)}$ we can now associate an element of $H^2_p(\mathcal{S}_4,\mathbb{Z})/(H^{(1,1)}_p(\mathcal{S}_4,\mathbb{R}) \cap H^2(\mathcal{S}_4,\mathbb{Z}))$, by forgetting about the always supersymmetric $(1,1)_p$ component of $B|_{\mathcal{S}_4}$.

Let us now see how this new moduli space $\mathcal{M}^F(\mathcal{S}_4)$ and its quantum numbers affect the spectrum of supersymmetric D7-branes. The BPS D7-branes with U(1) bundles can be classified in terms of the following topological quantities

(i) $[\mathcal{S}_4] \in H_4(\mathcal{M}_6,\mathbb{Z})$, such that $\mathcal{S}_4$ is holomorphic and $[H_3|_{\mathcal{S}_4}]$ is trivial.

(ii) $[F] \in H^{(1,1)}_p(\mathcal{S}_4,\mathbb{R}) \cap H^2(\mathcal{S}_4,\mathbb{Z})$

(iii) $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$

and to each topological sector we should attach the moduli space of $H^{0,1}(\mathcal{S}_4)$ Wilson lines.

Notice that the topological charges (i) and (ii) are already present in standard Calabi-Yau compactifications with closed B-field. Indeed, (i) corresponds to the homology class $[\mathcal{S}_4] \in H_4(\mathcal{M}_6,\mathbb{Z})$ of the four-cycle $\mathcal{S}_4 \subset \mathcal{M}_6$ that the D7-brane is wrapping, and (ii) to the Chern numbers $[F] \in H^2(\mathcal{S}_4,\mathbb{Z})$ of the U(1) gauge bundle field strength $F = dA$. On the other hand, $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$ is a new topological sector which arises after we introduce a background flux G_3, and that can be identified with a subset of $H^2_p(\mathcal{S}_4,\mathbb{Z})/(H^{(1,1)}_p(\mathcal{S}_4,\mathbb{R}) \cap H^2(\mathcal{S}_4,\mathbb{Z}))$.

It is important to notice that, in general, two D7-branes D7_α and D7_β in two different sectors of (i) and (ii) can be continuously connected to a third D7-brane D7_γ different from the previous ones. This can be done by simply giving a vev to the open strings living in the sector $\alpha\beta$, triggering the process of D-brane recombination $D7_\alpha + D7_\beta \to D7_\gamma$. Hence, in a given construction many of the vacua which differ by the open string quantum numbers (i) and (ii) are in fact connected by a continuous deformation. In addition, in some cases the final D7-brane is not described by a U(1) bundle but by a more general U(N) bundle, and such BPS D-brane is not even captured by the classification above.

One may wonder if something analogous could happen for the discretum of D7-brane positions $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$. In principle, since we have given a mass to the open strings that connect every two points of $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$, these fields cannot acquire a vev and each point of $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$ labels a truly disconnected vacuum. On the other hand, since our supersymmetry equations are based on the hypothesis of D7-branes with U(1) bundles, new supersymmetric solutions may arise when we consider $\mathcal{F}$ being the curvature of a U(N) bundle. Studying the spectrum of such D7-branes with U(N) bundles is beyond the scope of this paper. However, the computations performed in [16] for some simple cases suggest that the present results will generalize more or less directly to this more involved case.

Finally, one may wonder about the number of points contained in the lattice $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$, which is a question directly relevant for counting the number of open string vacua in a given compactification.[15] Presumably, the number of points in $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$ will grow exponentially with $h^{0,2}(\mathcal{S}_4)$, since this is the dimension of the lattice. For fixed $\mathcal{S}_4$ and $h^{0,2}(\mathcal{S}_4)$, we also expect the number of points of $\Lambda_{2h^{0,2}(\mathcal{S}_4)}$ to grow bigger for larger flux quanta, since then $B|_{\mathcal{S}_4}$ would vary more quickly on $\mathcal{M}(\mathcal{S}_4)$ and (4.20) would have more points. We indeed find such behavior for the toroidal example above. More precisely, from (4.16) we see that the amount of allowed locations for a D7-brane grows as N^2, which is proportional to the D3-brane tadpole introduced by the background flux $L = \int_{\mathcal{M}_6} \frac{g_s}{2} G_3 \wedge \bar{G}_3$. It would be interesting to see how this fact generalizes to more involved geometries. This suggests that the the number of open string vacua scales like

$$\mathcal{N}_{\text{open}} \sim L^{h^{0,2}}\,, \tag{4.21}$$

where L is the tadpole charge.

5. Applications

A geometrical understanding of D-brane moduli lifting and of the open string discretum, as the one provided via κ-symmetry above, should in principle give us new perspectives into some other issues related to D-branes and flux compactifications. The purpose of this section is to illustrate this fact by means of several direct applications of the results obtained above. We will first comment on the relevance of our results for the zero mode counting of type IIB D3-brane instantons. We will also explain how this geometrical framework of moduli stabilization may provide an alternative way of computing D-brane soft terms, as the ones that develop on the D7-brane effective theory in the presence of fluxes. Finally, we will point out some possible applications of our results to D7-brane model building.

5.1 Zero mode counting of D3-brane instantons

In the type IIB flux compactifications considered in this paper, the form of the perturbative superpotential leaves untouched the Kähler moduli of the Calabi-Yau. In order to get a rigid model, where no parameter can be continuously tuned, one must find a mechanism for fixing them. The proposal of [28] is that these moduli are fixed by a non-perturbative superpotential[16] generated by D-brane instantons.

The study of non-perturbative contributions in M/F-theory was initiated by Witten [29] for backgrounds without flux. [30 − 34] have revisited this important problem in the context where fluxes are present. In our work, we have found that open string moduli are generically lifted in the presence of fluxes. By supersymmetry, we expect that the corresponding fermionic zero modes also get lifted. In the case without fluxes the number

[15] Notice that the D-brane statistical analysis of ref.[27] deals exclusively with the topological sectors (i) and (ii), which can be connected to each other by giving vevs to fields. To the best of our knowledge, the effect of the disconnected, discrete positions of D7-branes has not been taken into account in the literature.

[16] If a suitable matter content is present on the D7-branes, one can also generate a non-perturbative superpotential by gaugino condensation.

of geometric zero modes is given by $h^{0,2}$, where $h^{0,2}$ is the Betti number of the four cycle that the instanton wraps. In the case with fluxes, the number of zero modes reduces to $h^{0,2}-r$, where r is the rank of the matrix defined in (3.14). Given that there are four-cycles that do not contribute to the superpotential in the case without flux because they have too many fermion zero modes, we expect that new contributions to the non-perturbative superpotential will arise in the case with fluxes. It would be interesting to analyze the problem in more detail.

5.2 Computing soft terms geometrically

Besides providing a general understanding of D7-brane moduli lifting, our techniques could also apply to compute the actual masses induced by the fluxes on the D7-brane worldvolume fields. A microscopic analysis of such effect was performed in [16], where the soft SUSY-breaking terms induced on a D7-brane were computed via dimensional reduction of the Myers' action. In order to illustrate the use of our approach in computing soft terms, let us reproduce some of the results in [16].

In principle, one should be able to compute the soft terms induced in the D7-brane bosonic sector by simply looking at the variation of its lagrangian density. Certain deformations ζ^i of the D7-brane would increase its energy, and this should be understood as a mass for the bosonic field Φ^i that corresponds to this deformation. In order to make contact with the computations in [16], let us consider a D7-brane wrapping $\mathcal{S}_4$, such that the pull-back $H_3|_{\mathcal{S}_4}$ vanishes identically and $\mathcal{F}=0$. By deforming $\mathcal{S}_4 \to \mathcal{S}_4'$, $\mathcal{F}$ may become a non-vanishing self-dual two-form. Then, after the computations in the appendix, one can see that the lagrangian density increases by

$$\delta\mathcal{L} = \int_{\mathcal{S}_4'} g_s^{-1}\Delta^{-2}\mathcal{F}^2 = 2\sum_k |a_k|^2 \int_{\mathcal{S}_4'} g_s^{-1}\Delta^{-2}\alpha^k\bar{\alpha}^k\,, \tag{5.1}$$

where we have decomposed $\mathcal{F}$ in terms of its self-dual components $\sum a_k\alpha^k+$c.c.. The mass matrix for the geometrical deformations $\{\zeta^i\}$ are then given by the second derivative of $\mathcal{L}$, that is

$$M^2_{i\bar{j}} = \frac{\partial\mathcal{L}}{\partial\zeta^i\partial\bar{\zeta}^{\bar{j}}} = 2\sum_k \frac{\partial a_k}{\partial\zeta^i}\overline{\frac{\partial a_k}{\partial\zeta^j}}\int_{\mathcal{S}_4} g_s^{-1}\Delta^{-2}\alpha^k\bar{\alpha}^k \tag{5.2}$$

and where the derivatives $\partial a_k/\partial\zeta^i$ are given by (3.14).

In order to simplify this expression, we can assume an homogeneous warp factor Δ and a constant dilaton, just as considered in [16]. We can then factorize the integral in (5.2), and the mass matrix $m^2_{i\bar{j}}$ becomes a simple quadratic function on the derivatives $\partial a_k/\partial\zeta^i$.

A particular simple case corresponds to a D7$_1$-brane on $(\mathbf{T}^2)^3$, just as in the toroidal example of the previous section. In this case the only geometric deformation is given by the position on the first $\mathbf{T}^2$ factor ζ^1, and the corresponding (0,2) form is given by $\iota_{\zeta^1}\Omega^{\mathbf{CY}} = dz^2\wedge dz^3$. In the presence of a (2,1) primitive flux G_3, the derivative of a_i is given by

$$\frac{\partial a_1}{\partial\zeta^1} = \frac{ig_s}{2}\int_{(\mathbf{T}^2)_2\times(\mathbf{T}^2)_3} \iota_{\zeta^1}\Omega^{\mathbf{CY}}\wedge\iota_{\zeta^1}\bar{G}_3 = \frac{ig_s}{2}\overline{G_{\bar{1}23}} \tag{5.3}$$

after the fields and forms have been conveniently normalized. Notice that the component $G_{\bar{1}23}$ is nothing but $\frac{1}{2}S_{\bar{1}\bar{1}}$ in terms of SU(3) irreducible representations. We then obtain that the mass of the adjoint scalar Φ^1 corresponding to the D7_1-brane deformation ζ^1 is given by

$$m^2_{1\bar{1}} = \frac{g_s}{8}|S_{1\bar{1}}|^2 \tag{5.4}$$

reproducing the result of [16].[17]

Although we have derived the mass of the geometrical modulus Φ^1 in a very simple case of the toroidal D7_1-brane, in principle the formula (5.2) can be applied to more involved geometries, with more than one geometric modulus and even a non-homogeneous warp factor. In addition, one should be able to derive the same kind of formulae for the more general case where $\mathcal{F}$ does not vanish. It would be very interesting to perform such computation and compare it to the results in [35, 17], where general soft-terms were computed by using effective action techniques.

5.3 D7-brane model building

Let us now discuss the relevance of our results to the construction of actual string models involving fluxes and D-branes. The fact that all D7-brane geometric moduli are typically lifted, and that there is a discrete set of choices for the vev's of these former moduli, will clearly open new possibilities for type IIB model building. Before addressing which these new possibilities may be, however, it proves useful to describe specific examples of BPS D7-branes in this class of compactifications.

In order to find such examples, let us first briefly recall some general features of the type IIB vacua under study. As already mentioned, a flux compactification yielding D=4 Poincaré invariance needs the presence of negative tension objects like O3$^-$-planes. The way to obtain O3-planes is to mod out a type IIB supergravity background by $\Omega^{ws}\mathcal{R}(-1)^{F_L}$. Here Ω^{ws} is the usual orientation reversal of the string world-sheet, whereas F_L is the space-time fermion number in the left-moving sector. Finally, $\mathcal{R}$ is an homolorphic involution which is also an isometry of the compact manifold $\mathcal{M}_6$, and which acts on the bispinor forms as [36]

$$\mathcal{R}J = J\,, \qquad \mathcal{R}\Omega = -\Omega\,, \tag{5.5}$$

and on the background fields as

$$\mathcal{R}B = -B, \qquad \mathcal{R}C_2 = -C_2\,. \tag{5.6}$$

Finally, the background field strengths $H_3 = dB$ and $F_3 = dC_2$ must be a sum of harmonic three-forms which are odd under the action of $\mathcal{R}$, just like Ω.

Let us now consider a D7-brane wrapping a four-cycle $\mathcal{S}_4$ invariant under $\mathcal{R}$, that is, $\mathcal{R}\mathcal{S}_4 = \mathcal{S}_4$. This may happen, e.g., when a D7-brane intersects an O3-plane. Let us assume that the pull-back $H_3|_{\mathcal{S}_4}$ is trivial in cohomology, and that every (2,0)-form $\alpha_2 \in H^{(2,0)}(\mathcal{S}_4)$ is even under the action of $\mathcal{R}|_{\mathcal{S}_4}$. It is then easy to see that such D7-brane

[17] There is actually a discrepancy in a factor of 9, due to a B-field gauge choice taken in [16] which is not globally well defined on the D7-brane worldvolume.

satisfies all the supersymmetry conditions. Because $\mathcal{S}_4$ is fixed by $\mathcal{R}$ it is automatically a complex submanifold of $\mathcal{M}_6$. In addition, since B is odd under the action of $\mathcal{R}$ and any $(2,0)$-form α_2 and J^{CY} are even, the integrals $\int_{\mathcal{S}_4} \alpha_2 \wedge B$ and $\int_{\mathcal{S}_4} J^{\text{CY}} \wedge B$ vanish. But these are exactly the conditions for the coefficients a_i and c in (3.7) to vanish, which is equivalent to require that a D7-brane is supersymmetric.

A particular case of the above consists in placing a D7-brane on top of an O7-plane. That is, we wrap the D7-brane around the four-cycle $\mathcal{S}_{O7}$ which is a fixed point locus of $\mathcal{R}$ (i.e., $\mathcal{R}z_0 = z_0\ \forall z_0 \in \mathcal{S}_{O7}$). In this case we can drop the mild conditions assumed above. Because H_3 is an odd three-form under the action of $\mathcal{R}$, it is easy to see that the pull-back $H_3|_{\mathcal{S}_{O7}}$ vanishes, not only in cohomology but identically. This is consistent with the fact that, due to (5.6), the field B must vanish on $\mathcal{S}_{O7}$. Since B vanishes, we can always choose $\mathcal{F} = 0$, which clearly satisfies all the supersymmetry requirements.

To summarize, we find that all the additional supersymmetry conditions introduced by type IIB three-form fluxes are automatically satisfied when D7-branes are placed on top of an O7-plane,[18] and (under some mild assumptions) when they intersect an O3-plane. The $\mathcal{N} = 1$ D7-branes are then characterized by the same choices of $\mathcal{F}$ as if the background flux G_3 was not present.

Let us now consider which class of D7-branes may be more suitable for constructing viable models of particle physics and/or cosmology. In principle, a very interesting possibility consist of D7-branes wrapping a four-cycle $\mathcal{S}_4$ such that $H^{0,1}(\mathcal{S}_4) = 0$. Indeed, by Poincaré duality $H^3(\mathcal{S}_4) = 0$, and any such D7-brane is automatically free of Freed-Witten anomalies. In addition, the moduli space of Wilson lines is zero dimensional, so the only possible moduli are the geometric deformations of $\mathcal{S}_4$. However, we have seen that these moduli are generically lifted by the presence of the background flux G_3 so, at the end of the day, this gauge sector of the theory is free of open string moduli.

The absence of D7-brane moduli translates in a U(N) low energy theory without massless fields in the adjoint.[19] This is quite an attractive feature for constructing viable models of particle physics and, in particular, semi-realistic models yielding asymptotic freedom [37]. On a different spirit, one can think of using these D7-branes to build confining hidden sectors which, via gaugino condensation, generate a non-perturbative superpotential for Kähler moduli [14].[20]

Eventually, one may conceive building flux compactifications with semi-realistic chiral sectors arising from open strings stretched between D7-branes. A simple example of such construction was given in [38], where an MSSM-like model was constructed by using three sets of D7-branes, wrapped on four-cycles $\mathcal{S}_4^a$, $\mathcal{S}_4^b$ and $\mathcal{S}_4^c$ of a $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold background. The gauge group arising from such D7-branes is the SU(4) $\times$ SU(2) $\times$ SU(2)

[18]That $\mathcal{S}_{O7}$ defines a supersymmetric four-cycle of $\mathcal{M}_6$ was somehow to be expected, since an $O7$-plane is part of the closed string/supergravity background, and our working hypothesis in this paper is that such background preserves $D = 4$ $\mathcal{N} = 1$ supersymmetry.

[19]Strictly speaking, the absence of massless adjoint fields is a stronger requirement that the absence of moduli, but it also generically satisfied.

[20]Notice that an Euclidean D3-brane wrapping such moduli-free four-cycle would also contribute to a non-perturbative superpotential for Kähler moduli.

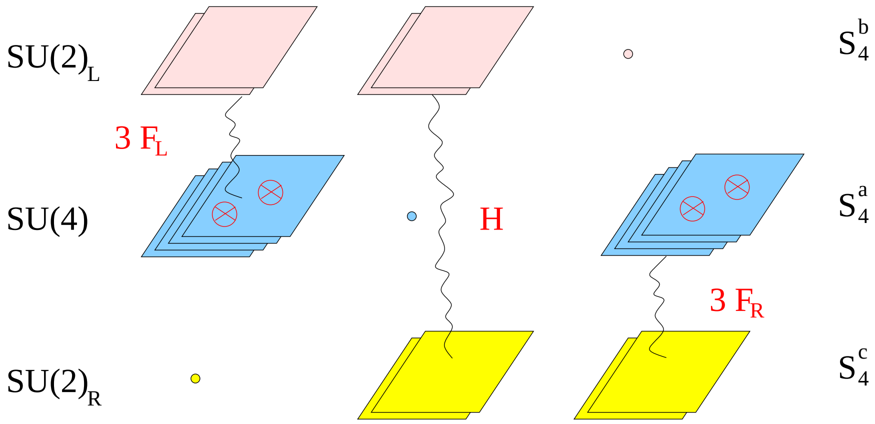

Figure 2: Pati-Salam D7-brane model. The open strings with both ends on the same D-brane yield the gauge group $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, while those ending on different yield the bifundamental matter $F_L = (4, 2, 1)$, $F_L = (\bar{4}, 1, 2)$ and $H = (1, 2, 2)$. Chirality and family replication are due to a non-trivial worldvolume flux $F = dA$ on the SU(4) D7-brane.

Pati-Salam extension of the Standard Model, and a simple choice of gauge bundle on the SU(4) D7-brane provides the desired $\mathcal{N} = 1$ chiral spectrum with 3 fermion generations and a minimal Higgs sector. See figure 2 for a schematic representation of this model.

This construction is remarkably simple and, in principle, one can extend it to more general metric backgrounds by choosing three four-cycles $\mathcal{S}_4^a$, $\mathcal{S}_4^b$ and $\mathcal{S}_4^c$ with triple intersection number $\mathcal{S}_4^a \cap \mathcal{S}_4^b \cap \mathcal{S}_4^c = 1$ and appropriate gauge bundles. However, in order to discuss realistic particle physics one needs to connect this Pati-Salam theory with the Standard Model. The usual approach is to perform the adjoint Higgsing $\mathrm{SU}(4) \to \mathrm{SU}(3) \times \mathrm{U}(1)$ which, in compactifications without background G_3 fluxes, is a flat direction of the theory. Now, the field Φ which acquires a vev is nothing but a massless adjoint field of the theory, so this mechanism is not entirely satisfactory from a phenomenological point of view.

The existence of a D7-brane discretum, however, adds new possibilities for adjoint Higgsing. If our SU(4) D7-brane is placed in a ISD flux background, one can realize the Pati-Salam breaking $\mathrm{SU}(4) \to \mathrm{SU}(3) \times \mathrm{U}(1)$ by 'discrete' adjoint Higgsing, just by first considering 4 D7-branes located on top of each other and then placing one of them in a different supersymmetric location. Notice that the former flat direction is a geometric modulus which is now lifted by the presence of the flux, so Φ is now given by a massive adjoint field whose vev is at the minimum of a scalar potential. This indeed mimics the field theory mechanism for Pati-Salam adjoint breaking.

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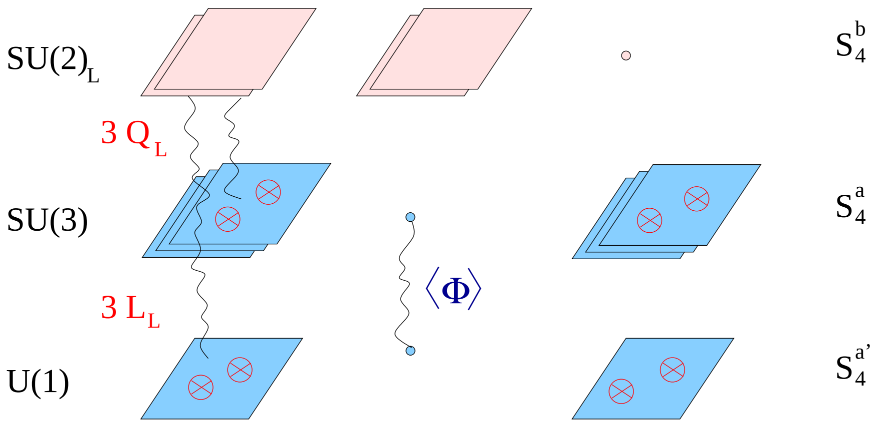

Figure 3: Pati-Salam breaking SU(4) → SU(3) × U(1) by 'discrete' adjoint Higgsing.

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A. D7-brane energetics

A non-trivial check of the supersymmetry conditions derived from κ-symmetry can be performed by considering the lagrangian density associated to a D7-brane. Indeed, if we are moving along the moduli space of a supersymmetric D-brane, its lagrangian density (i.e., its action) should remain constant. On the other hand, if we deform our D-brane in such a way that we break supersymmetry, in general we would expect that the total energy of such D-brane increases after the deformation.

Let us see how this works for a D7-brane in presence of a flux background. The bosonic part of a D7-brane effective action is given by

$$S_{D7} = - \int_{M_4 \times \mathcal{S}_4} g_s^{-1} \sqrt{|g + \mathcal{F}|} - \int_{M_4 \times \mathcal{S}_4} \sum_n C_{2n} \wedge e^{\mathcal{F}} , \tag{A.1}$$

where g is the induced metric on the D7-brane worldvolume, which is wrapping the four-cycle $\mathcal{S}_4$, and $\sum_n C_{2n}$ is the usual sum of C_{2n} RR potentials, $n = 0, \ldots, 4$, which are completed with powers of $\mathcal{F}$ up to an 8-form.

As we have seen in subsection 2.1, the fact that a D-brane satisfies the κ-symmetry conditions allow us to express its contribution to the DBI part of the action by a simple set of p-forms evaluated over the D-brane worldvolume. In particular, for the case of a D7-brane, wrapping a holomorphic four-cycle $\mathcal{S}_4$ and having an anti-selfdual (ASD) field strength $\mathcal{F}$ allows to write (A.1) as

$$\begin{aligned} S_{D7} &= - \int_{M_4 \times \mathcal{S}_4} \Phi_{DBI} - \int_{M_4 \times \mathcal{S}_4} \Phi_{WZ} \\ &= - \int_{M_4 \times \mathcal{S}_4} g_s^{-1} \Delta^{-2} \mathrm{dvol}_{M_4} \wedge \frac{1}{2} \left(J^2 - \mathcal{F}^2 \right) - \int_{M_4 \times \mathcal{S}_4} \sum_n C_{2n} \wedge e^{\mathcal{F}} , \end{aligned} \tag{A.2}$$

where $\Delta^{-2}d\mathrm{vol}_{M_4} = \Delta^{-2}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ is the contribution to the DBI energy from the non-compact dimensions wrapped by the D7-brane. It is clear that if we deform our D7-brane embedding as $\mathcal{S}_4 \to \mathcal{S}_4'$, such that $\mathcal{S}_4'$ is still holomorphic and $\mathcal{F}$ is an ASD two-form in $\mathcal{S}_4'$, the expression (A.2) will not change, and now we should evaluate $\Phi_{DBI} + \Phi_{WZ}$ on the worldvolume $M_4 \times \mathcal{S}_4'$ in order to compute S_{D7}. Since $\mathcal{S}_4$ and $\mathcal{S}_4'$ both belong to the same homology class $[\mathcal{S}_4] \in H_4(\mathcal{S}_4, \mathbb{Z})$, the only thing that we need to check in order to show that S_{D7} does not change is that $\Phi_{DBI} + \Phi_{WZ}$ is a closed form. That is,

$$d\,(\Phi_{DBI} + \Phi_{WZ}) \;=\; 0 \tag{A.3}$$

so that S_{D7} will not change when we move inside $[\mathcal{S}_4]$ and, at the same time, we respect the κ-symmetry conditions.

Let us see that this is indeed the case. Let us first recall that type IIB flux compactifications with warped ansatz (2.12), $G_{n\overline{m}} = \Delta(Z) G^{CY}_{n\overline{m}}$ and O3$^-$-planes require that

(i) We turn on a 5-form field strength $\tilde{F}_5 = (1 + *_{10})\, d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$, related to the warp factor by $d\alpha = d(\Delta^{-2} g_s^{-1})$.

(ii) G_3 is an imaginary self-dual (ISD) 3-form of $\mathcal{M}_6$, i.e., $*_6 G_3 = iG_3$.

For simplicity, we will assume a background constant axion-dilaton $\tau = C_0 + i/g_s$. The Wess-Zumino contribution to the D7-brane action and its differential are given by

$$\Phi_{WZ} \;=\; C_8 + C_6 \wedge \mathcal{F} + \frac{1}{2} C_4 \wedge \mathcal{F}^2 + \frac{1}{3!} C_2 \wedge \mathcal{F}^3 + \frac{1}{4!} C_2 \wedge \mathcal{F}^4 \tag{A.4}$$

$$d\Phi_{WZ} \;=\; \tilde{F}_9 + \tilde{F}_7 \wedge \mathcal{F} + \frac{1}{2} \tilde{F}_5 \wedge \mathcal{F}^2 + \frac{1}{3!} \tilde{F}_3 \wedge \mathcal{F}^3 + \frac{1}{4!} \tilde{F}_1 \wedge \mathcal{F}^4\,, \tag{A.5}$$

where

$$\tilde{F}_{2n+1} = dC_{2n} + H_3 \wedge C_{2n-2} \tag{A.6}$$

are the generalized field strengths of type IIB supergravity, related to each other by $\tilde{F}_{2n+1} = *_{10} \tilde{F}_{9-2n}$. Using these duality relations and the fact that $\mathcal{F}$ has support on the four-cycle $\mathcal{S}_4$, we arrive at

$$\begin{aligned} d\Phi_{WZ} \;&=\; *_{10} \tilde{F}_3 \wedge \mathcal{F} + \frac{1}{2} \mathrm{dvol}_{M_4} \wedge d\alpha \wedge \mathcal{F}^2 \\ &=\; \frac{1}{2} \mathrm{dvol}_{M_4} \wedge d\left(\Delta^{-2} g_s^{-1} \mathcal{F}^2\right), \end{aligned} \tag{A.7}$$

where in the last equality we have used that the supergravity background satisfies $*_6 G_3 = iG_3$ and $d\alpha = d(\Delta^{-2} g_s^{-1})$.

It is now easy to compute the differential of $\Phi_{DBI} + \Phi_{WZ}$

$$\begin{aligned} d\,(\Phi_{DBI} + \Phi_{WZ}) \;&=\; \frac{1}{2} \mathrm{vol}_{M_4} \wedge d\left[g_s^{-1} \Delta^{-2} (J^2 - \mathcal{F}^2 + \mathcal{F}^2)\right] \\ &=\; \frac{1}{2} g_s^{-1} \mathrm{vol}_{M_4} \wedge d(J^{\mathrm{CY}})^2\,, \end{aligned} \tag{A.8}$$

where we have used the explicit expression of the two-form $J = \Delta\, J^{\rm CY}$ in terms of the unwarped Calabi-Yau Kähler form $J^{\rm CY}$, and the fact that g_s is constant.[21] Clearly (A.8) vanishes, because $J^{\rm CY}$ is a closed form. Hence we obtain that S_{D7} does not change if the κ-symmetry conditions are satisfied.

On the other hand, when we break the supersymmetry conditions we would expect the D7-brane action to increase in value. Indeed, notice that whenever $\mathcal{F}$ is not anti-selfdual $J^2 - \mathcal{F}^2$ does no longer compute the tension associated to the four-cycle $\mathcal{S}_4$. In particular, if we consider a self-dual $\mathcal{F} = *_4\mathcal{F}$, such tension would be given by the integral of

$$\sqrt{|g^{\mathcal{S}_4} + \mathcal{F}|} = \frac{1}{2}\left(J^2 + \mathcal{F}\wedge *_4\mathcal{F}\right)\big|_{\mathcal{S}_4} = \frac{1}{2}\left(J^2 - \mathcal{F}^2\right)\big|_{\mathcal{S}_4} + \mathcal{F}\wedge *_4\mathcal{F} \geq \frac{1}{2}\left(J^2 - \mathcal{F}^2\right)\big|_{\mathcal{S}_4} \quad \text{(A.9)}$$

with the equality being saturated only for $\mathcal{F} = 0$.[22] Let us then consider a D7-brane undergoing the deformation $\mathcal{S}_4 \to \mathcal{S}_4'$, with vanishing $\mathcal{F}$ at $\mathcal{S}_4$ and a self-dual two-form $\mathcal{F}$ at $\mathcal{S}_4'$. By performing similar computations as above, it is easy to see that the change in the bosonic action is given by

$$\begin{aligned} S_{D7} - S_{D7}' &= \int_{M_4\times\mathcal{S}_4'} \Phi_{DBI} + \Phi_{WZ} + g_s^{-1}\Delta^{-2}\text{dvol}_{M_4}\wedge\mathcal{F}^2 - \int_{M_4\times\mathcal{S}_4} \Phi_{DBI} + \Phi_{WZ} \\ &= \int_{\mathcal{S}_4'} g_s^{-1}\Delta^{-2}\mathcal{F}^2\,, \end{aligned} \quad \text{(A.10)}$$

where in the second line we have normalized the action with respect to the non-compact dimensions, and we have used the fact that $\Phi_{DBI} + \Phi_{WZ}$ is a closed form.

Notice that the computations above remind of those in [15], where it was performed an analysis of supersymmetric D-branes by means of generalized calibrations. Indeed, in the notation of [15], $\Phi_{DBI} + \Phi_{WZ}$ would be the generalized calibration for D7-branes on a compactification with fluxes, and the supersymmetric D7-branes would be the ones that are calibrated with respect to it. As we know, the fact that $\Phi_{DBI} + \Phi_{WZ}$ is a calibration not only implies that it is closed, but also that any D-brane which is not calibrated by it will have higher energy than the calibrated ones. In the present case, D7-branes wrapping holomorphic four-cycles and with an anti-selfdual two-form $\mathcal{F}$ are the ones that minimize their energy, as we would expect from the fact that they are supersymmetric.

It is important to see that, from the point of view of κ-symmetry, the supersymmetry conditions for a type IIB D-brane are given in terms of the non-closed forms Ω and J defined on a general SU(3)-structure manifold, rather than by a generalized calibration. As advanced in [15] and checked here, both approaches are not in contradiction, but rather are complementary approaches to characterize BPS D-branes in SU(3)-structure compactifications. In fact, the κ-symmetry equations derived in subsection 2.1 are a non-trivial refinement of the proposal made in [15].

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[21] Notice that, had we not assumed a constant axion-dilaton $\tau = C_0 + i/g_s$, then we should also have introduced new sources on the Wess-Zumino part of the action $\int \Phi_{WZ}$.

[22] In general, we can prove that the inequality $\sqrt{|g^{\mathcal{S}_4} + \mathcal{F}|} \geq \frac{1}{2}\left(J^2 - \mathcal{F}^2\right)\big|_{\mathcal{S}_4}$ is only saturated by $\mathcal{S}_4$ being holomorphic and $\mathcal{F}$ anti-selfdual.

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